What can solvable Hatsugai-Kohmoto model teach us about Non-Fermi Liquid

Yin Zhong

Lanzhou University

zhongy@lzu.edu.cn

HKU-UCAS Yong Physicists Symposium 2024
August 23, 2024

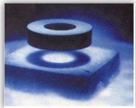
Outline

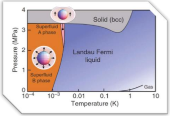
Motivation

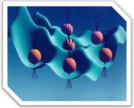
Key question in condensed matter physics:

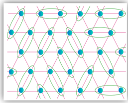
What is the ground state of the interacting electron system?

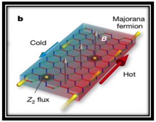
- *Symmetry breaking states: SC, SDW, CDW, DDW, nematic, stripe, loop current...*
- *No symmetry breaking states: FL, TLL, NFL...*
- *topological states: FQH, QSL...*











Motivation

What is the ground state of the interacting electron system? Landau's answer:

Fermi liquid

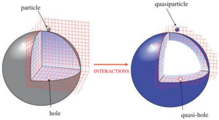
weakly interacting Fermi gas with renormalized parameters (mass, magnetic moment...)

Key point of FL: Adiabatic principle

- Increase interaction gradually before phase transition, dressed particle called quasiparticle
- Non-interacting Fermi gas evolves into interacting Fermi liquid.
- FL is the standard model for He-3, metal, heavy fermion...

$$|\Psi_{FG}\rangle = \prod_{k<k_F,\sigma=\uparrow,\downarrow} |n^0_{k\sigma}\rangle \rightarrow |\Psi_{FL}\rangle = \prod_{k<k_F,\sigma=\uparrow,\downarrow} |n_{k\sigma}\rangle \tag{1}$$

From perspective of RG, FL is a stable fixed point for spatial dimension larger than one. (except for BCS pairing)



Motivation

Just recall some consequence of Fermi liquid.

Thermodynamics:

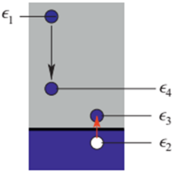
- specific heat: $C_V = \gamma^* T, \gamma^* = \frac{\pi^2}{3} N^*(0)$
- charge susceptibility: $\chi_c = \frac{\partial n}{\partial \mu} = \frac{N^*(0)}{1+F_0^s}$
- spin susceptibility: $\chi_s = \frac{\partial M}{\partial B} = \frac{N^*(0)}{1+F_0^a} \mu_B$

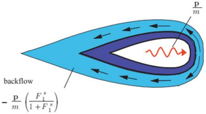
Quasiparticle:

- effective mass: $\frac{m^*}{m} = 1 + F_1^s$
- scattering rate: $\frac{1}{\tau_k} \sim \frac{1}{E_F}((\varepsilon_k - E_F)^2 + (\pi T)^2) \Rightarrow \rho(T) \sim T^2$

Collective mode excitation:

- zero sound mode: $\omega = s v_F^* q$, $\frac{s}{2} \ln \frac{s+1}{s-1} - 1 = \frac{1}{F_0^s}$





Motivation

Microscopic justification of FL:

Luttinger's theorem

density of electron equals to volume of Fermi sphere[Luttinger, 1960]

$$n = 2\int \frac{d^d k}{(2\pi)^d}\theta(\mathrm{Re}G(k,\omega=0)) = \frac{\mathcal{V}_{FS}}{(2\pi)^d} \tag{2}$$

- We can use Green function's pole to find Fermi surface
- It is valid for Fermi gas, Fermi liquid and Luttinger liquid
- Know Fermi surface=know density of electron

The proof of Luttinger's theorem is based on the validity of Luttinger-Ward functional formalism.[Luttinger,Ward 1960] Also see,

- Oshikawa's topological argument[Oshikawa, 2000]
- determinant formalism[Seki, 2017]

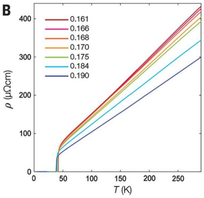
Motivation

If those FL's predictions are not consistent with experiments or theories, this is the sign of breakdown of Fermi liquid!

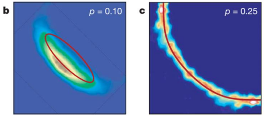
non-Fermi liquid

They are called non-Fermi liquid if they are still metallic without any explicit symmetry breaking but show behaviors deviating from FL.

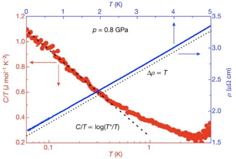
- Linear-T or power-law resistivity: $\rho \sim T$ or $\rho \sim T^{\alpha}$, $0 < \alpha < 2$. (e.g. $\alpha = 3/2$ in $CeNi_2Ge_2$)
- Anomalous specific heat: $C_v \sim T \ln T$ or $C_v \sim T^{\beta}$. (e.g. $\beta = 2/3$ in $YbRi_2Si_2$)
- Breakup of closed Fermi surface into Fermi arc. (cuprate)



Resistivity of cuprate



Fermi arc of cuprate

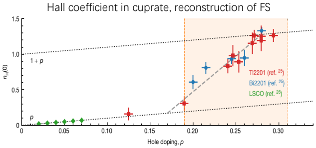


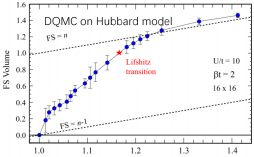
CeRh6Ge4, a FM heavy fermion

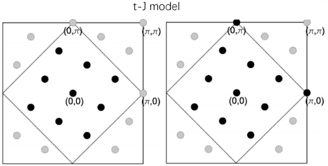
Motivation

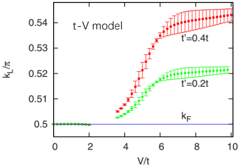
Violation of Luttinger's theorem:

- Hall coefficient in cuprate.
- DQMC simulation in Hubbard model.[Osborne,Paiva,Trivedi, 2021] (other studies in $t-V$ and $t-J$ models[Kokalj,Prelovsek, 2007, Kokalj,Prelovsek, 2008])







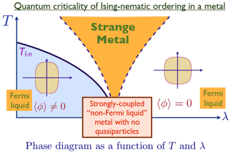


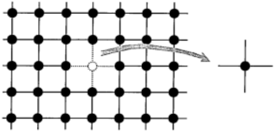
Motivation

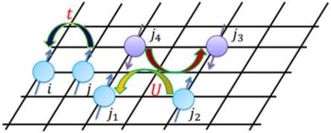
What is the theory of non-Fermi liquid?

1. Fermi surface+X: X=order parameter, gauge-field (RPA,perturbative RG)
2. DMFT: map lattice into impurity (pesudogap Anderson impurity model)[Georges et al., 1996]
3. Solvable models with infinite-ranged interaction: SY, SYK, HK[Chowdhury et al., 2022]

In this talk, we follow the third road and focus on the so-called HK model.







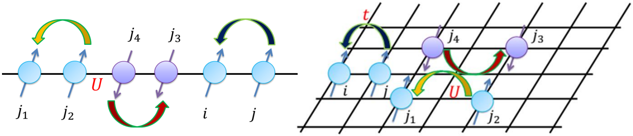
HK model

Motivated by infinite-ranged spin glass SY model,[Sherrington,Kirkpatrick, 1975] Hatsugai and Kohmoto propose the following model in 1992[Hatsugai,Kohmoto, 1992]

Hatsugai-Kohmoto model

$$H_{HK} = -t \sum_{\langle i,j,\sigma \rangle} c^\dagger_{i\sigma} c_{j\sigma} - \mu \sum_{j\sigma} c^\dagger_{j\sigma} c_{j\sigma} + \frac{U}{N_s} \sum_{j_1,j_2,j_3,j_4} \delta_{j_1+j_3=j_2+j_4} c^\dagger_{j_1\uparrow} c_{j_2\uparrow} c^\dagger_{j_3\downarrow} c_{j_4\downarrow} \tag{3}$$

- With infinite-ranged interaction, preserves center of mass
- Exact solution in any dimension and any electron filling
- Solution with NFL and Mott insulator
- Similar infinite-ranged $t-J$ model is invented by Baskaran in 1991[Baskaran, 1991]



Solve HK model

- How to solve HK: Fourier transformation $c_{j\sigma} = \frac{1}{\sqrt{N_s}} \sum_k e^{ikR_j} c_{k\sigma}$.

$$\frac{U}{N_s} \sum_{j_1,j_2,j_3,j_4} \delta_{j_1+j_3=j_2+j_4} c^\dagger_{j_1\uparrow} c_{j_2\uparrow} c^\dagger_{j_3\downarrow} c_{j_4\downarrow} \Rightarrow U \sum_k n_{k\uparrow} n_{k\downarrow} \tag{4}$$

due to conservation of center of matter and the infinite-ranged interaction.

Diagonalized in momentum space:

$$H = \sum_k H_k = \sum_k [(\varepsilon_k - \mu)(n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k\downarrow}] \tag{5}$$

- Easy to solve for each k: eigenstate and eigenenergy

1. $|0\rangle_k, E_0(k) = 0$
2. $|\uparrow\rangle_k = c^\dagger_{k\uparrow}|0\rangle_k, E_u(k) = \varepsilon_k - \mu$
3. $|\downarrow\rangle_k = c^\dagger_{k\downarrow}|0\rangle_k, E_n(k) = \varepsilon_k - \mu$
4. $|\uparrow\downarrow\rangle_k = c^\dagger_{k\uparrow} c^\dagger_{k\downarrow}|0\rangle_k, E_d(k) = 2(\varepsilon_k - \mu) + U$

Solve HK model

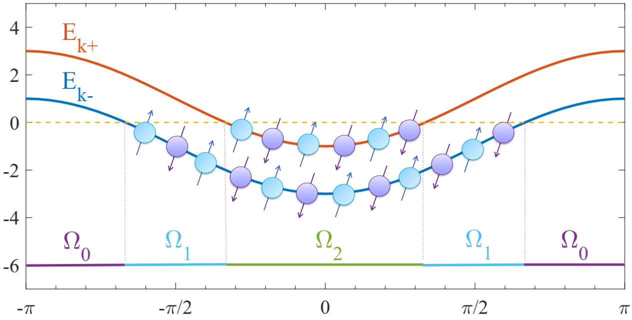
- Many-body eigenstates: product of each k-state

$$|\Psi\rangle = \prod_{k\in\Omega_0} |0\rangle_k \prod_{k\in\Omega_1} |\alpha =\uparrow,\downarrow\rangle_k \prod_{k\in\Omega_2} |\uparrow\downarrow\rangle_k, \tag{6}$$

and its eigenenergy is

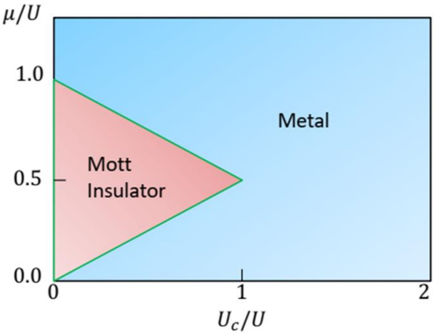
$$E = \sum_{k\in\Omega_1} (\varepsilon_k - \mu) + \sum_{k\in\Omega_2} (2\varepsilon_k - 2\mu + U) = \sum_{k\in\Omega_1} E_{k-} + \sum_{k\in\Omega_2} (E_{k-} + E_{k+}). \tag{7}$$

1. Ω_0: no occupation regime, $|0\rangle_k$
2. Ω_1: single occupation regime, $|\uparrow\rangle_k, |\downarrow\rangle_k$
3. Ω_2: double occupation regime, $|\uparrow\downarrow\rangle_k$



Phase diagram of HK model

If all *k*-states are single-occupied, the system is in Mott insulator, otherwise it is in non-Fermi-liquid-like metal.

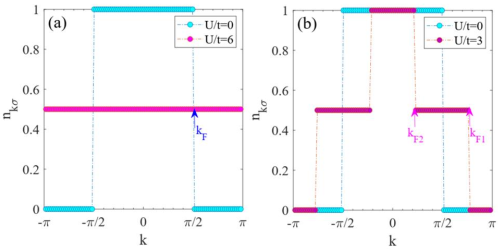


The boundary of Mott phase is determined by (setting excitation energy to zero)

$$\frac{\mu}{U} = 1 - \frac{W}{2U}; \quad \frac{\mu}{U} = \frac{W}{2U}; \quad W/U = 0 \tag{8}$$

But how about its non-Fermi liquid nature?

Non-Fermi-liquid-like metal: electron's distribution function



- Two jumps in nk: two pseudo-Fermi surfaces

$$n_{k\sigma}^{T=0} = \frac{1}{2}\left[\theta(\mu + 2t\cos k) + \theta(\mu + 2t\cos k - U)\right] \quad (9)$$

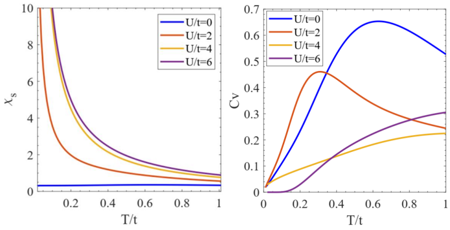
$$k_{F2} = \left|\arccos\frac{U-\mu}{2t}\right|, \quad k_{F1} = \left|\arccos\frac{-\mu}{2t}\right|. \quad (10)$$

- Count particle density: violation of Luttinger's theorem

$$n = \overbrace{2k_{F2}}^{\Omega_2}\frac{2}{2\pi} + \overbrace{2(k_{F1} - k_{F2})}^{\Omega_1}\frac{2}{2\pi}\frac{1}{2} = (k_{F1} + k_{F2})\frac{2}{2\pi} \neq \overbrace{2k_{F2}\frac{2}{2\pi} + 2k_{F1}\frac{2}{2\pi}}^{\text{required by Luttinger's theorem}} \quad (11)$$

HK model

Non-Fermi-liquid-like metal: non-Pauli susceptibility and linear-T specific heat.



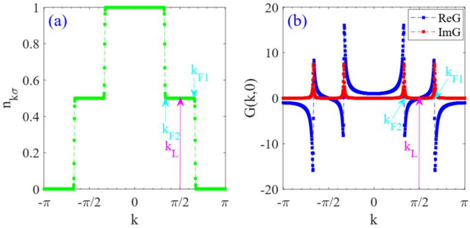
- Curie-like spin susceptibility in NFL, because single occupation means spin-$1/2$ local moment and it contributes $1/T$ spin susceptibility.
- Why Curie but not Pauli? More interesting explanation from exclusion statistics of Haldane![Haldane, 1991]

$$W = \prod_{i,\mu} \frac{[D_i^{\mu}(\{N_j^{\nu}\}) + N_i^{\mu} - 1]!}{N_i^{\mu}![D_i^{\mu}(\{N_j^{\nu}\}) - 1]!}, \quad D_i^{\mu}(\{N_j^{\nu}\}) + \sum_{j,\nu} g_{ij}^{\mu\nu} N_j^{\nu} = G_i^{\mu} \tag{12}$$

- Gas of particles with exclusion statistics.
$G_c = 1, G_s = 0, g_{cc} = g_{ss} = 1, g_{sc} = -1, g_{cs} = 0$.[Hatsugai et al., 1996]

Non-Fermi-liquid-like metal: single particle excitation

$$G_\sigma(k,\omega) = \frac{1 - \langle n_{k\bar{\sigma}} \rangle}{\omega - (\varepsilon_k - \mu)} + \frac{\langle n_{k\bar{\sigma}} \rangle}{\omega - (\varepsilon_k - \mu + U)} \tag{13}$$



- Luttinger's surface: zeros of $G_\sigma(k, \omega = 0)$ with Luttinger wavevector k_L.

Breakdown of Luttinger theorem if $\mu \neq U/2$, (particle-hole symmetry enforces Luttinger theorem at $\mu = U/2$)

$$2 \int \frac{dk}{2\pi} ReG(k,0) = 2k_L \frac{2}{2\pi} \neq n \tag{14}$$

- Non-Landau's quasiparticles: holon and doublon

$$h_{k\sigma} = c_{k\sigma}(1-n_{k\bar{\sigma}}), \quad d_{k\sigma} = c_{k\sigma}n_{k\bar{\sigma}} \tag{15}$$

Non standard anticommutative rule,

$$[d_{k\sigma}, d^{\dagger}_{k'\sigma'}]_{+} = \delta_{kk'}\delta_{\sigma\sigma'}n_{k\bar{\sigma}}, [h_{k\sigma}, h^{\dagger}_{k'\sigma'}]_{+} = \delta_{kk'}\delta_{\sigma\sigma'}(1-n_{k\bar{\sigma}}) \tag{16}$$

So, holon and doublon are not standard fermions. Direct calculation based on Feynman diagrams and Wick theorem must be breakdown.

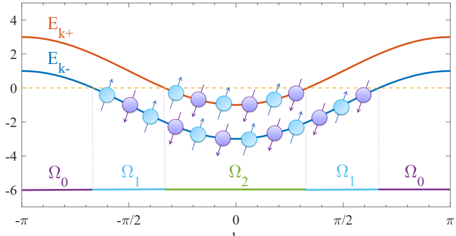
- Exact Green's function:

$$\langle\langle h_{k\sigma}|h^{\dagger}_{k\sigma}\rangle\rangle = \frac{1-\langle n_{k\bar{\sigma}}\rangle}{\omega-(\varepsilon_k-\mu)} = \frac{1-\langle n_{k\bar{\sigma}}\rangle}{\omega-E_{k-}}, \quad \langle\langle d_{k\sigma}|d^{\dagger}_{k\sigma}\rangle\rangle = \frac{\langle n_{k\bar{\sigma}}\rangle}{\omega-(\varepsilon_k-\mu+U)} = \frac{\langle n_{k\bar{\sigma}}\rangle}{\omega-E_{k+}} \tag{17}$$

Note,

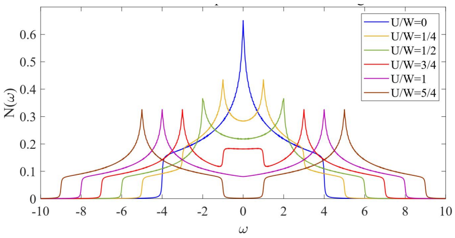
$$G_{\sigma}(k,\omega) = \langle\langle c_{k\sigma}|c^{\dagger}_{k\sigma}\rangle\rangle = \langle\langle h_{k\sigma}|h^{\dagger}_{k\sigma}\rangle\rangle + \langle\langle d_{k\sigma}|d^{\dagger}_{k\sigma}\rangle\rangle \tag{18}$$

Although holon and doublon are not standard fermions, they are the quasi-particle of HK model and contribute to the bands E_{k-}, E_{k+}.



Density of state of metal-Mott transition:

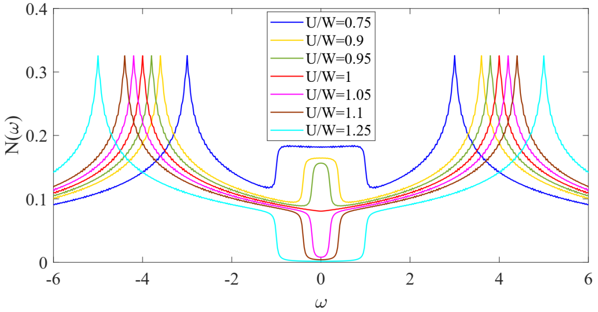
$$N(\omega) = -\frac{1}{\pi}\sum_k \mathrm{Im}G_\sigma(k, \omega + i0^+) \tag{19}$$



- $U < W$(bandwidth): no gap in metallic state
- $U > W$: has full gap in Mott state
- There are quasi-particle-like density of state around Fermi energy $\omega = 0$ if U is near bandwidth from metallic state.

DOS of HK model

More details of density of state of metal-Mott transition (symmetric half-filling):



- Gradually vanished quasi-particle density around Fermi energy $\omega = 0$.
- Similar to the structure of single-site DMFT of Hubbard model. But HK case is due to band-filling without any Kondo effect.

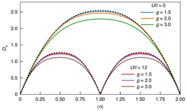
$$G_{Hubbard} \simeq \frac{Z}{\omega + iT_K} + \frac{1/2 - Z/2}{\omega - \varepsilon_k + U/2} + \frac{1/2 - Z/2}{\omega - \varepsilon_k - U/2} \tag{20}$$

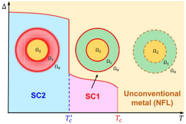
$$G_{HK} = \frac{1/2}{\omega - \varepsilon_k + U/2} + \frac{1/2}{\omega - \varepsilon_k - U/2} \tag{21}$$

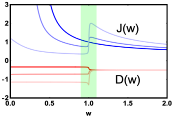
Extension of HK model

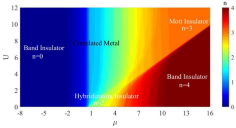
There are many extension of HK model:

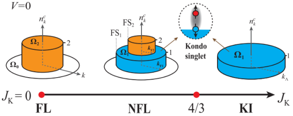
- BCS superconductivity[Phillips et al., 2020, Li et al., 2022, Zhu et al., 2021]
- Kondo impurity[Setty, 2021]
- Fermi arc with anisotropic HK interaction[Yang, 2021]
- Heavy fermion[Zhong, 2022, Wang et al., 2024]
- Topological/Chern insulator[Wysokinski, 2023, Mai et al., 2023]











Extension of HK model:PAM-HK

The periodic Anderson model (PAM) is a standard model for heavy fermion systems but cannot be exactly solved by usual analytical or numerical techniques.

$$
\begin{aligned}
H_{PAM} &= -\sum_{i,j,\sigma} t_{ij}^{c} c_{i\sigma}^{\dagger} c_{j\sigma} - \sum_{i,j,\sigma} t_{ij}^{f} f_{i\sigma}^{\dagger} f_{j\sigma} + E_f \sum_{j\sigma} f_{j\sigma}^{\dagger} f_{j\sigma} + V \sum_{j\sigma} (c_{j\sigma}^{\dagger} f_{j\sigma} + h.c.) \\
&- \mu \sum_{j\sigma} (c_{j\sigma}^{\dagger} c_{j\sigma} + f_{j\sigma}^{\dagger} f_{j\sigma}) + U \sum_{j} f_{j\uparrow}^{\dagger} f_{j\uparrow} f_{j\downarrow}^{\dagger} f_{j\downarrow} \qquad (22)
\end{aligned}
$$

We find the extension of HK, the PAM-HK model can be solved, which replaces Hubbard with HK interaction[Zhong, 2022]

$$
\begin{aligned}
H_{PAM-HK} &= -\sum_{i,j,\sigma} t_{ij}^{c} c_{i\sigma}^{\dagger} c_{j\sigma} - \sum_{i,j,\sigma} t_{ij}^{f} f_{i\sigma}^{\dagger} f_{j\sigma} + E_f \sum_{j\sigma} f_{j\sigma}^{\dagger} f_{j\sigma} + V \sum_{j\sigma} (c_{j\sigma}^{\dagger} f_{j\sigma} + h.c.) \\
&- \mu \sum_{j\sigma} (c_{j\sigma}^{\dagger} c_{j\sigma} + f_{j\sigma}^{\dagger} f_{j\sigma}) + \frac{U}{N_s} \sum_{j_1, j_2, j_3, j_4} \delta_{j_1+j_3=j_2+j_4} f_{j_1\uparrow}^{\dagger} f_{j_2\uparrow} f_{j_3\downarrow}^{\dagger} f_{j_4\downarrow} \qquad (23)
\end{aligned}
$$

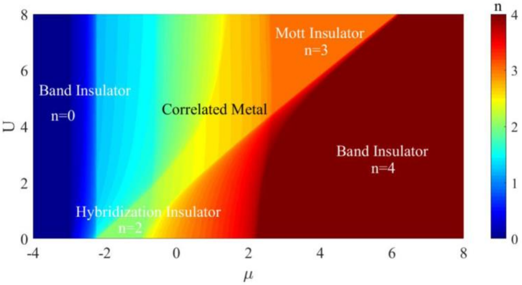
Use FT, $H = \sum_k H_k$,

$$
\begin{aligned}
H_k &= \sum_{\sigma} (\varepsilon_k^c - \mu) c_{k\sigma}^{\dagger} c_{k\sigma} + \sum_{\sigma} (\varepsilon_k^f + E_f - \mu) f_{k\sigma}^{\dagger} f_{k\sigma} \\
&+ V \sum_{\sigma} (c_{k\sigma}^{\dagger} f_{k\sigma} + f_{k\sigma}^{\dagger} c_{k\sigma}) + U f_{k\uparrow}^{\dagger} f_{k\uparrow} f_{k\downarrow}^{\dagger} f_{k\downarrow}, \qquad (24)
\end{aligned}
$$

It can be solved by diagonalizing 16×16-matrix for each k.

Extension of HK model:PAM-HK

The ground-state phase diagram is



There are four states in the ground-state:

1. Band insulator, $n = 0, 4$, gapped due to empty/full occupation
2. Correlated metal, gapless, violates Luttinger's theorem
3. Hybridization/Kondo insulator, $n = 2$, gapped due to hybridization between c and f
4. Mott insulator, $n = 3$, gapped due to band splitting of strong correlation

The trivial band insulator:

- Particle density $n = 0, 4$ ($n = \frac{1}{N_s}\sum_k n_k$) correspond to empty or full occupation for each k-state.
- The wave-functions are $\prod_k |0000\rangle_k, \prod_k |1111\rangle_k$. ($|c\uparrow, c\downarrow, f\uparrow, f\downarrow\rangle$)

Hybridization insulator:

- Particle density $n = 2$, dominated by hybridization strength V.

HI can be understood from $U = 0$ limit, and one has the following quasi-particle Hamiltonian

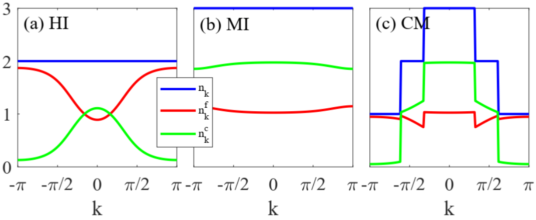
$$H_k = \sum_\sigma (E_{k+}\alpha^\dagger_{k\sigma}\alpha_{k\sigma} + E_{k-}\beta^\dagger_{k\sigma}\beta_{k\sigma}) \tag{25}$$

- The quasi-particle energy is $E_{k\pm} = \frac{1}{2}(\varepsilon_k + E_f \pm \sqrt{(\varepsilon_k - E_f)^2 + 4V^2}) - \mu$, it gives two-band structure
- $\alpha_{k\sigma} = \mu_k c_{k\sigma} + \nu_k f_{k\sigma}, \beta_{k\sigma} = -\nu_k c_{k\sigma} + \mu_k f_{k\sigma}$

If lower band E_{k-} is fully occupied, but upper band E_{k+} is empty, HI (with ground-state wave-function $\prod_{k\sigma}\hat{\beta}^\dagger_{k\sigma}|0\rangle$) with particle density $n = 2$ appears, which has the indirect gap $\sim V$ and direct gap $\Delta \equiv \min(E_{k+}) - \max(E_{k+}) \sim \frac{V^2}{t}$.

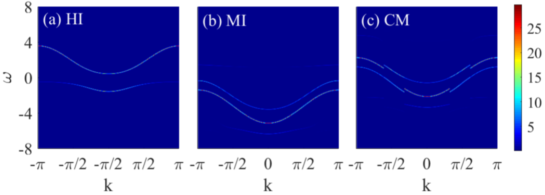
Extension of HK model:PAM-HK

HI, MI and CM have different particle distribution:



- HI has $n_k = 2$ for each k
- MI has $n_k = 3$ for each k
- CM has two jumps, corresponding to two Fermi surface. It may have only one jump if Fermi energy is too positive or negative.

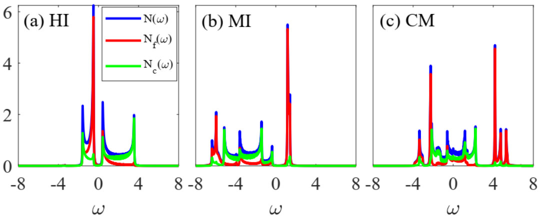
Spectral function:



- HI has two bands and direct gap
- MI has four bands and three bands below the Fermi energy
- CM has four bands, but no direct gap around the Fermi energy

Extension of HK model:PAM-HK

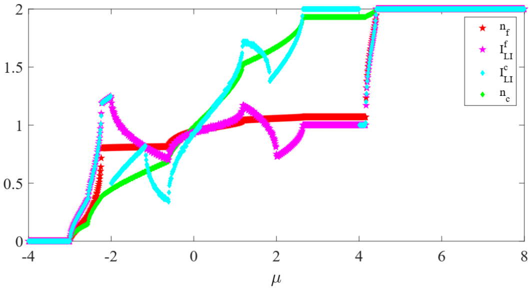
Density of state:



- HI has small gap
- MI has sensible gap
- No gap around Fermi energy in CM

Extension of HK model:PAM-HK

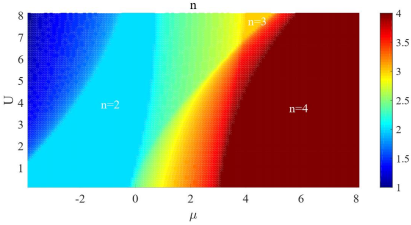
Violation of Luttinger's theorem in CM:

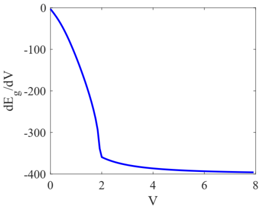


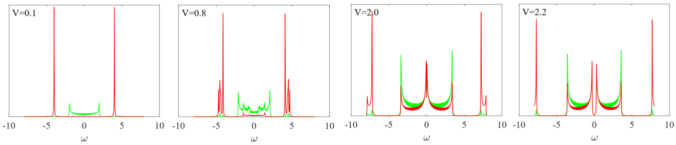
$$I_{LI} = \sum_{\sigma} \int \frac{dk}{2\pi} \theta(\mathrm{Re}G_{\sigma}(k, \omega = 0)). \tag{26}$$

- CM is a non-Fermi liquid state, violating Luttinger's theorem. This is similar to the case of HK model, where the single occupation regime appears.

For symmetric half-filling case, ($E_f = -4, U = 8, \mu = 0$), we change V and there is a transition from CM to HI.



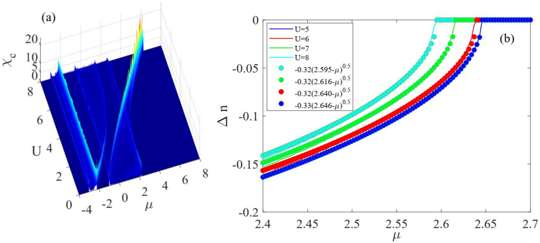




When V is small, for f-electron only local energy level E_f and $E_f + U$ is visible. Increasing V leads to the appearance of DOS around Fermi energy. When V is large, there is f-electron band around Fermi energy and this band opens a gap if V is further increased.

Extension of HK model:PAM-HK

Phase transition between CM and MI is Lifshitz transition since observable satisfy scaling behaviors.



$$\Delta n = n - n_0 \sim (\mu - \mu_c)^{\beta}, \;\; \beta = d/2 = 1/2 \tag{27}$$

- Other scaling behaviors $E_g - E_g^0 \sim (\mu - \mu_c)^{(d+2)/2}$, $\chi_c \sim (\mu - \mu_c)^{(d-2)/2}$

Relation between particle density and metallic state: a Lieb-Schultz-Mattis (LSM) argument.

Consider periodic boundary condition and define the twist operator

$$U = e^{i\sum_{j=1}^{N_s} \frac{2\pi j}{N_s} \sum_\sigma (c_{j\sigma}^\dagger c_{j\sigma} + f_{j\sigma}^\dagger f_{j\sigma})}. \tag{28}$$

If we denote the ground-state as $|\Psi_0\rangle$, then a new state is constructed by applying U, i.e. the twisted state $U|\Psi_0\rangle$. So, one can calculate the energy difference

$$\begin{aligned} \Delta E &= \langle\Psi_0|U^{-1}HU|\Psi_0\rangle - \langle\Psi_0|H|\Psi_0\rangle \\ &= \sum_\sigma \sum_{j=1}^{N_s} (2 - e^{-i2\pi/N_s} - e^{i2\pi/N_s}) t\langle c_{j\sigma}^\dagger c_{j+1\sigma}\rangle \sim \mathcal{O}(1/N_s) \end{aligned} \tag{29}$$

There exists at least one low-energy state near ground-state. Furthermore, for the translation operator T, we have $TUT^{-1} = Ue^{-i2\pi n}$. ($n = \frac{1}{N_s}\sum_{j\sigma}(c_{j\sigma}^\dagger c_{j\sigma} + f_{j\sigma}^\dagger f_{j\sigma})$) Assume the ground-state $|\Psi_0\rangle$ has particle density n and momentum P_0, then

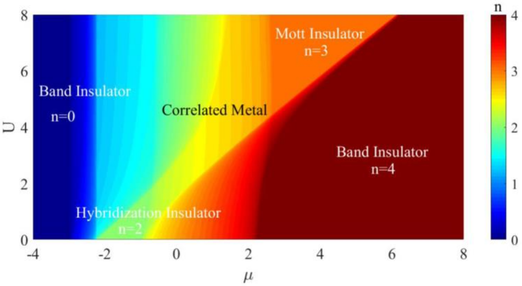
$$TU|\Psi_0\rangle = UTe^{-i2\pi n}|\Psi_0\rangle = e^{-i2\pi n} e^{-iP_0} U|\Psi_0\rangle, \tag{30}$$

Extension of HK model:PAM-HK

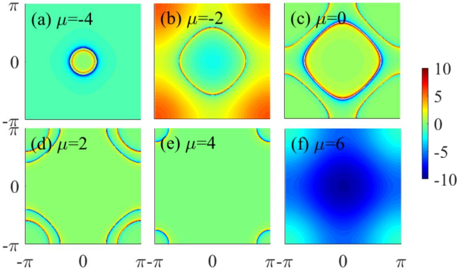
$$TU|\Psi_0\rangle = e^{-i2\pi n} e^{-iP_0} U|\Psi_0\rangle, \tag{31}$$

- Twisted state $U|\Psi_0\rangle$ is the eigenstate of momentum $2\pi n + P_0$.
- If n is not an integer, $U|\Psi_0\rangle$ and $|\Psi_0\rangle$ must be orthogonal, thus the system is gapless and it corresponds to metallic state.
- When n is an integer, $U|\Psi_0\rangle$ and $|\Psi_0\rangle$ are the same state, it suggests there exist no low-energy state and the system should be an insulator.

Therefore, we now understand why $n = 0, 2, 3, 4$ in our model correspond to insulating states and generic electron's density implies metallic state.



In $d = 2$ square lattice, the phase diagram is similar to the $d = 1$ case. Its Fermi surface is determined by maximum of spectral function $A(k, 0)$ and $\mathrm{Re}G(k, 0)$.



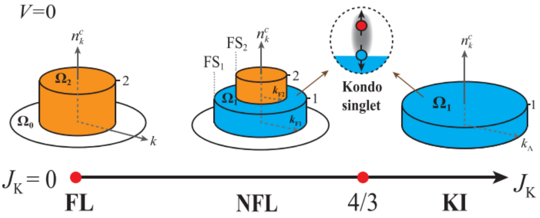
The above figure plots $\mathrm{Re}G(k, 0)$. The multiple Fermi surface structure is visible. The mismatch of Fermi surface implies the appearance of Luttinger surface. These metallic phases violate Luttinger's theorem.

More implication of our PAM-HK model:

- Use Schrieffer-Wolf transformation, the PAM-HK model can be mapped into HK-Kondo lattice model

$$H = \sum_{k\sigma}(\varepsilon_k - \mu)c^\dagger_{k\sigma}c_{k\sigma} + J\sum_k \vec{s}_k \cdot \vec{S}_k \tag{32}$$

This model has been solved by Wang, Li and Yang.



It is interesting to see the Kondo insulator, whose wavefunction and Green's function are

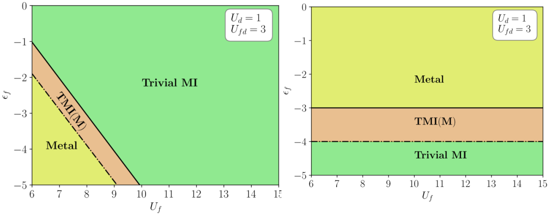
$$|\Psi_{KI}\rangle = \prod_k \frac{1}{\sqrt{2}}(|\uparrow,\Downarrow\rangle_k - |\downarrow,\Uparrow\rangle_k). \; G(k,\omega) = \frac{1/2}{\omega - \varepsilon_k - 3J/4} + \frac{1/2}{\omega - \varepsilon_k + 3J/4}. \tag{33}$$

More implication of our PAM-HK model:

- When spin-orbit coupling is included, one has a topological PAM-HK model.

$$H = \sum_k \left[\Psi_k^\dagger \begin{pmatrix} \varepsilon_k - \mu & V_k \\ V_k & E_f - \mu \end{pmatrix} \Psi_k + U_f n^f_{k\uparrow} n^f_{k\downarrow} + U_{cf} n^c_k n^f_k + U_c n^c_{k\uparrow} n^c_{k\downarrow} \right] \tag{34}$$

$V_k = -2V(\sin k_x \sigma_x + \sin k_y \sigma_y)$. This model has been solved.[Jablonowski et al., 2023]



There exist topological Mott insulators for $n = 1$ and $n = 3$. But no such state for $n = 2$, so exact solution for topological Kondo insulator is still lacking.

Extension of HK model:FO

Friedel oscillation (FO) in metal: non-magnetic impurity immerses in Fermi sea, a regular modulation of charge density around impurity appears.

- How to find FO theoretically? Add impurity potential.

$$H_{\mathrm{imp}} = V\sum_{\sigma} c_{0\sigma}^{\dagger}c_{0\sigma} = \frac{V}{N_{\mathrm{s}}}\sum_{k,k',\sigma} c_{k\sigma}^{\dagger}c_{k'\sigma}.$$

Use T-matrix approximation,

$$\begin{aligned} G_{\sigma}(k,k',\omega) &\simeq \delta_{k,k'}G_{\sigma}(k,\omega) + G_{\sigma}(k,\omega)T_{\sigma}(\omega)G_{\sigma}(k',\omega) \\ T_{\sigma}(\omega) &= \frac{V/N_{\mathrm{s}}}{1-VF_{\sigma}(\omega)}, \qquad F_{\sigma}(\omega) = \frac{1}{N_{\mathrm{s}}}\sum_{k} G_{\sigma}(k,\omega). \end{aligned} \tag{35}$$

$$\delta n_i = \frac{1}{N_{\mathrm{s}}}\sum_{k,k',\sigma} \exp(i(k-k')R_i) \times \int d\omega f_{\mathrm{F}}(\omega)\frac{-1}{\pi}\mathrm{Im}\delta G_{\sigma}(k,k',\omega). \tag{36}$$

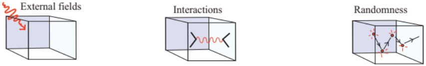
For Fermi liquid and Luttinger liquid, ($2k_F$ oscillation)

$$\delta n_i \equiv n_i - n \sim \frac{\cos(2k_{\mathrm{F}}|R_i|)}{|R_i|^{g}}, \qquad |R_i| >> 1, \;\; g = \begin{cases} d, & \text{FL;} \\ K, & \text{TL.} \end{cases} \tag{37}$$

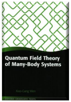
Alternative formalism, the linear response theory,

$$\langle \delta \hat{\mathcal{O}}(t) \rangle = \frac{1}{i\hbar} \int_{-\infty}^{t} dt' f(t') \langle [\hat{\mathcal{O}}_H(t), \hat{\mathcal{O}}_H(t')] \rangle = \int_{-\infty}^{\infty} dt' \chi(t,t') f(t')$$

Kubo formula: how observable related to external forc



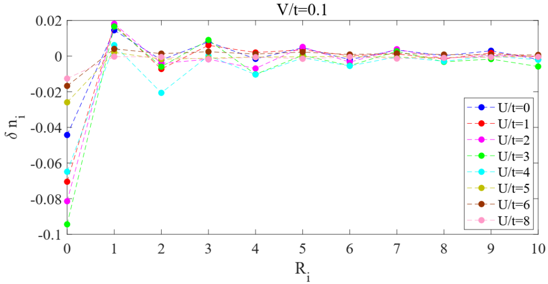
What is physics? ~response of system



$$\begin{aligned} \delta n_i(t) &= \frac{1}{i} \int_{-\infty}^{t} dt' \langle [n_i(t), H_{\mathrm{imp}}(t')] \rangle = \frac{1}{i} \int_{-\infty}^{\infty} dt' \theta(t-t') \langle [n_i(t), n_0(t')] \rangle V(t') \\ &= -\int_{-\infty}^{\infty} dt' \chi_{\mathrm{c}}(R_i, R_0, t-t') V(t') \end{aligned}$$

Extension of HK model:FO

We find HK model has FO,[Zhao et al., 2023]

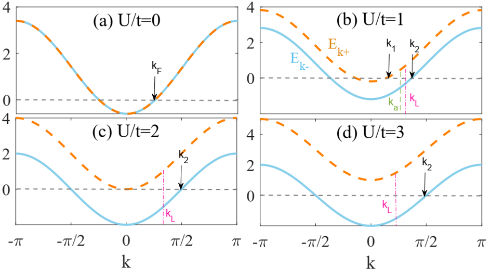


But why it has FO?

- For FL and TL, FO is a result from $2k_F$ singularity.

Extension of HK model:FO

For HK, it has multiple Fermi surface,

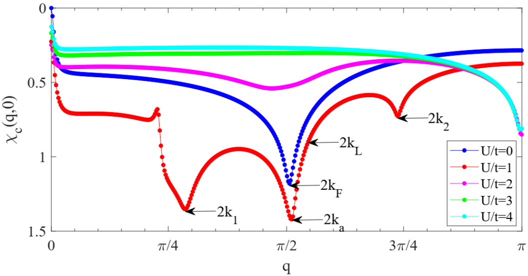


$$\delta n_i \sim \cos(2k_1(R_i - R_0))\chi_c(2k_1) + \cos(2k_2(R_i - R_0))\chi_c(2k_2) + \cos(2k_a(R_i - R_0))\chi_c(2k_a) \quad (38)$$

- Main contributions to δn_i come from intra-band and inter-band transition involving $2k_1$, $2k_2$ and $2k_a = k_1 + k_2$ momentum transfer.

Extension of HK model:FO

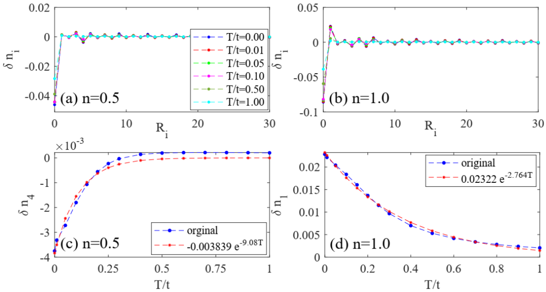
- The donimating contribution is the inter-band transition involving $2k_a = k_1 + k_2$ momentum transfer.$\delta n_i \sim \cos(2k_\mathrm{a}(R_i - R_0))\chi_\mathrm{c}(2k_\mathrm{a})$



$$
\begin{aligned}
\chi_\mathrm{c}(q, i\Omega_n) \quad &= \quad \frac{-1}{N_\mathrm{s}} \sum_{k,\sigma} (1 - n_k)(1 - n_{k+q}) \times \frac{f_\mathrm{F}(\varepsilon_k - \mu) - f_\mathrm{F}(\varepsilon_{k+q} - \mu)}{i\Omega_n - \varepsilon_{k+q} + \varepsilon_k} \\
&+ \quad \frac{-1}{N_\mathrm{s}} \sum_{k,\sigma} (1 - n_k) n_{k+q} \times \frac{f_\mathrm{F}(\varepsilon_k - \mu) - f_\mathrm{F}(\varepsilon_{k+q} - \mu + U)}{i\Omega_n - \varepsilon_{k+q} - U + \varepsilon_k} \\
&+ \quad \frac{-1}{N_\mathrm{s}} \sum_{k,\sigma} n_k (1 - n_{k+q}) \times \frac{f_\mathrm{F}(\varepsilon_k - \mu + U) - f_\mathrm{F}(\varepsilon_{k+q} - \mu)}{i\Omega_n - \varepsilon_{k+q} + \varepsilon_k + U}
\end{aligned}
$$

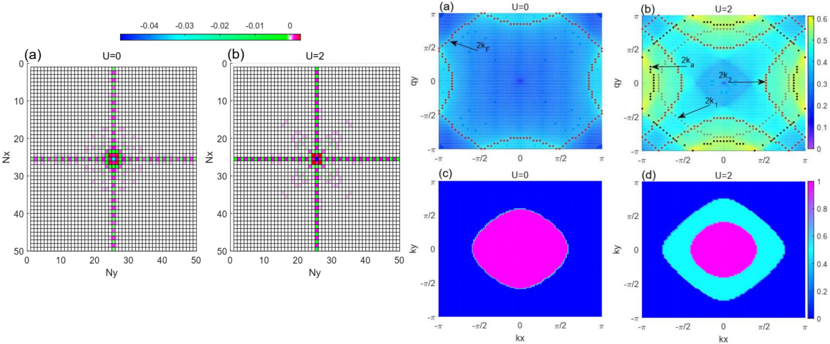
Extension of HK model:FO

Finite-T effect, suppression of FO.



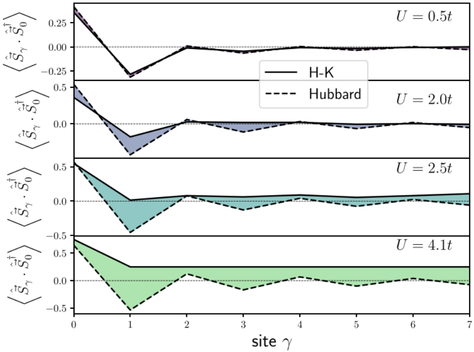
The amplitude of FO has exponential decay versus temperature T.

FO in $d = 2$, still dominated by inter-band transition.



Extension of HK model:FO

FO has also been studied by real space ED.[Skolimowski, 2024] For OBC, one finds a long-ranged ferromagnetic correlation at strong coupling in contrast to antiferromagnetic correlation in Hubbard model.



It is interesting to see whether the FM correlation is a finite-size effect when the system is larger.

Conclusion

Let me summarize the main point of this talk:

- We have given an introduction to HK model and focused on its non-Fermi liquid nature. The key point is the appearance of single-occupation regime, which leads to the breakdown of Luttinger's theorem and non-Fermi liquid behavior.
- Extension of HK, e.g. the PAM-HK model has been solved and its phase diagram is uncovered. This model has been related to HK-Kondo lattice and topological PAM.
- FO of HK is analyzed and it is dominated by inter-band transition. ED study in real space is interesting since FM correlation is found.

To proceed, scattering effect of holon and doublon should be included, which will lead to finite resistivity and may be more relevant to experiments in strongly correlated systems.

Another point is that, if the interaction is short-ranged but conserves the center of mass, can we still have the properties of Hk-like models? It seems that this problem may be answered by numerical simulation such as MPS or tensor-network.

References

J. M. Luttinger (1960)
Fermi Surface and Some Simple Equilibrium Properties of a System of Interacting Fermions
Phys. Rev. 119, 1153.

J. M. Luttinger and J. C. Ward (1960)
Ground-State Energy of a Many-Ferraion System. II
Phys. Rev. 118, 1417.

M. Oshikawa (2000)
Topological Approach to Luttingers Theorem and the Fermi Surface of a Kondo Lattice
Phys. Rev. Lett. 84, 3370.

K. Seki and S. Yunoki (2017)
Topological interpretation of the Luttinger theorem
Phys. Pev. B 96, 085124.

I. Osborne, T. Paiva and N. Trivedi (2021)
Broken Luttinger theorem in the two-dimensional Fermi-Hubbard model
Phys. Pev. B 104, 235122.

J. Kokalj and P. Prelovšek (2007)
Luttinger sum rule for finite systems of correlated electrons
Phys. Pev. B 75, 045111.

J. Kokalj and P. Prelov̌sek (2008)
Breakdown of the Luttinger sum rule within the Mott-Hubbard insulator
Phys. Pev. B 78, 153103.

A. Georges, G. Kotliar, W. Krauth and M. J. Rozenberg (1996)
Dynamical mean-field theory of strongly correlated fermion systems and the limit of infinite dimensions
Rev. Mod. Phys. 68, 13.

D. Chowdhury, A. Georges, O. Parcollet and S. Sachdev (2022)
Sachdev-Ye-Kitaev models and beyond: Window into non-Fermi liquids
Rev. Mod. Phys. 94, 035004.

Y. Hatsugai and M. Kohmoto (1992)
Exactly Solvable Model of Correlated Lattice Electrons in Any Dimensions
J. Phys. Soc. J. 61, 2056.

G. Baskaran (1991)
An Exactly Solvable Fermion Model:Spinons,Holons and a Non-Fermi liquid phase
Mod. Phys. Lett. B 5, 643.

F. D. M. Haldane (1991)
"Fractional Statistics" in Arbitrary Dimensions: A Generalization of the Pauli Principle
Phys. Rev. Lett. 67, 937.

Y. Hatsugai, M. Kohmoto, T. Koma and Y.-S. Wu (1996)
Mutual-exclusion statistics in exactly solvable models in one and higher dimensions at low temperatures
Phys. Pev. B 54, 5358.

R. Wang and K. Yang (2023)
Non-Fermi liquid behavior in a simple model of Fermi arcs and pseudogap
Mod. Phys. Lett. B 37, 2350119.

J. Zhao, G. La Nave and P. W. Phillips (2023)
Proof of a stable fixed point for strongly correlated electron matter
Phys. Pev. B 108, 165135.

J. Skolimowski (2024)
Real-space analysis of Hatsugai-Kohmoto interaction
Phys. Pev. B 109, 165129.

P. W. Phillips, L. Yeo, and E. W. Huang (2020)
Exact theory for superconductivity in a doped Mott insulator
Nat. Phys. 16, 1175.

Y. Li, V. Mishra, Y. Zhou, and F.-C. Zhang (2022)
Two-stage superconductivity in the HatsugaiC Kohomoto-BCS model
New J. Phys. 24,103019.

H.-S. Zhu, Z. Li, Q. Han, and Z. D. Wang (2021)
Topological s-wave superconductors driven by electron correlation
Phys. Rev. B 103,024514.

C. Setty (2021)
Dilute magnetic moments in an exactly solvable interacting host
arXiv 2105.15205.

K. Yang (2021)
Exactly solvable model of Fermi arcs and pseudogap
Phys. Rev. B 103,024529.

Y. Zhong (2022)
Solvable periodic Anderson model with infinite-range Hatsugai-Kohmoto interaction: Ground-states and beyond
Phys. Rev. B 106, 155119.

J. Wang, Y. Li, and Y.-f. Yang (2024)
Simple solvable model for heavy-fermion superconductivity from the two-fluid normal state
Phys. Rev. B 109, 064517.

M. M. Wysokinski, and W. Brzezicki (2023)
Quantum anomalous Hall insulator in ionic Rashba lattice of correlated electrons
Phys. Rev. B 108, 035121.

P. Mai, B. E. Feldman, and P. W. Phillips (2023)
Topological Mott insulator at quarter filling in the interacting Haldane model
Phys. Rev. Res. 5,013162.

M. Zhao, W.-W. Yang, H.-G. Luo, and Y. Zhong (2023)
Friedel oscillation in non-Fermi liquid: lesson from exactly solvable HatsugaiCKohmoto model
J. Phys.: Condens. Matter 35,495603.

D. Sherrington and S. Kirkpatrick (1975)
Solvable Model of a Spin-Glass
Phys. Rev. Lett. 35,1792.

K. Jablonowski, J. Skolimowski, W. Brzezicki, K. Byczuk, and M. M. Wysokinski (2023)
Solvable Model of a Spin-Glass
Phys. Rev. B 108, 195145.

G. D. Mahan (2000)
Many-Particle Physics
Springer Science+Business Media New York.

The End

Questions? Comments?